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Partnership with Partial Commitment

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Abstract

This paper derives the Nash-equilibrium degrees of commitment to a partnership where lack of full commitment fuels suspicion and increases potential losses for partners.

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1. Introduction

The term partial commitment is usually used in game-theoretic and bargaining studies to indicate a player's revocable obligation to a position on a partition of a cake (e.g., Muthoo, 1996; Calem, 1997; and Henkel, 2002). In this paper it is used to describe a partner's degree of devotion to her partnership and disregard of external options. As frequently happens in business, politics and interpersonal relationships, attractive external options are realized after a partnership is formed. The effects of such external options on the rational choice of degrees of commitment to a partnership are conceptually analyzed in this paper. In particular, the paper considers the case where the direct returns on the external options exceed the internal returns, but high pecuniary and non-pecuniary costs prevent the partner(s) with the external option(s) from quitting (separation) and render partial commitment, though exposable, a viable strategy. The paper suggests that the Nash-equilibrium commitment of each member of a one-shot partnership (or, more generally, a known finite number of times automatically renewed partnership) increases with her internal return, quitting cost and costs of being deserted and with her counterpart's external return and intrinsic capacity to detect lack of full commitment, but diminishes with her external return and intrinsic capacity to detect her counterpart's lack of full commitment and with her counterpart's internal return, quitting cost and costs of being deserted.

The paper is structured as follows. Section 2 describes the effects of partial commitment on the partners' internal returns, external returns, potential losses and detection of each other's lack of full commitment. Section 3 presents the partner's payoffs and strategies. Section 4 derives the Nash-equilibrium degrees of commitment in a one-period partnership. Section 5 illustrates the effects of the model parameters on the partners' Nash-equilibrium degrees of commitment with numerical simulations.

2. External and internal returns, potential losses and detection capacities

I and *J* are partners. With similar notations for *J*, the degree of *I*'s commitment to her partnership with *J* is denoted by $0 < \theta_i \leq 1$, where $\theta_i = 1$ indicates full commitment, $0 < \theta_i < 1$ partial commitment, and $1 - \theta_i$ *I*'s degree of devotion to her external option.

Suppose, that the external return for *I* increases, linearly for simplicity, with $1 - \theta_i$:

$$z_i = (1 - \theta_i)\hat{z}_i \tag{1}$$

where $\hat{z}_i = \lim_{\theta_i \to 0} z_i$ indicates the external return when *I* quits the partnership and is fully devoted to her external option. Suppose, similarly, that *I*'s gross internal return is proportional to her degree of commitment to the partnership:

$$y_i = \theta_i \hat{y}_i \tag{2}$$

where $\hat{y}_i = \lim_{\theta_i \to 1} y_i$ indicates the gross internal return for *I* when *I* is fully committed to the partnership.

Internal return for a partner is depreciated by the exposure of her own lack of full commitment and by her realization of her counterpart's lack of full commitment. Naturally, the higher *I*'s degree of commitment the lower the probability (p_j) of *I*'s lack of full commitment being detected by *J* and also the lower the loss (L_i^i) for *I* from *J*'s discovery of *I*'s lack of full commitment. This assumption is displayed, for simplicity, by a linear specification of the detection probability

$$p_j = (1-\theta_i)\hat{p}_j\,,\;\; 0 < \hat{p}_j \le 1 \tag{3}$$

and by a linear specification of the potential loss

$$L_i^i = (1-\theta_i)\hat{L}_i^i\,,\;\; \hat{L}_i^i > 0\,. \tag{4}$$

In this framework, $\hat{p}_j = \lim_{\theta_i \to 0} p_j$ and $\hat{L}_i^i = \lim_{\theta_i \to 0} L_i^i$ and hence can be interpreted as *J*'s intrinsic capacity to detect *I*'s lack of full commitment and the maximum potential loss for *I* when *I* quits (the quitting cost), respectively. Similarly, the probability (p_i) of *J*'s lack of full commitment being detected by *I* and the loss (L_j^i) for *I* from discovering *J*'s lack of full commitment are taken to be linearly diminishing in *J*'s degree of commitment:

$$p_i = (1-\theta_j)\hat{p}_i\,,\;\; 0 < \hat{p}_i \le 1 \tag{5}$$

and

$$L_j^i = (1-\theta_j)\hat{L}_j^i\,,\;\; \hat{L}_j^i > 0\,. \tag{6}$$

Here, $\hat{p}_i = \lim_{\theta_i \to 0} p_i$ is *I*'s intrinsic capacity to detect *J*'s lack of full commitment and $\hat{L}_j^i = \lim_{\theta_i \to 0} L_j^i$ is the loss for *I* when *J* quits – the costs for *I* from being deserted by *J*.

The partners' intrinsic capacities to detect each other's lack of full commitment are not necessarily equal: $\hat{p}_j > \hat{p}_i$ means that *J* is less susceptible to deception than *I*, and *vice versa*. If *I*'s ego is strong and *I*'s integrity is weak her quitting cost is smaller than the costs of being deserted ($\hat{L}_i^i < \hat{L}_j^i$) and, in view of the above assumptions, *I*'s loss from the exposure of her lack of full commitment is smaller than *I*'s loss from realizing an identical lack of commitment of her counterpart.

3. Payoffs and strategies

In view of the uncertainty about being exposed as partially committed and the uncertainty about *J*'s degree of commitment, the net return for partner *I* (x_i) is a random variable distributed as follows[1]

$$x_i = \begin{cases} \theta_i \hat{y}_i + (1-\theta_i)(\hat{z}_i - \hat{L}_i^i) - (1-\theta_j)\hat{L}_j^i & (1-\theta_i)\hat{p}_j(1-\theta_j)\hat{p}_i \\ \theta_i \hat{y}_i + (1-\theta_i)(\hat{z}_i - \hat{L}_i^i) & (1-\theta_i)\hat{p}_j \\ \theta_i \hat{y}_i + (1-\theta_i)\hat{z}_i - (1-\theta_j)\hat{L}_j^i & (1-\theta_j)\hat{p}_i \\ \theta_i \hat{y}_i + (1-\theta_i)\hat{z}_i & 1-(1-\theta_i)\hat{p}_j(1-\theta_j)\hat{p}_i - (1-\theta_i)\hat{p}_j - (1-\theta_j)\hat{p}_i \end{cases} \quad .(7)$$

The expected net returns for *I* and *J* are:

$$E(x_i) = \theta_i \hat{y}_i + (1-\theta_i)\hat{z}_i - (1-\theta_i)^2 \hat{p}_j[(1-\theta_j)\hat{p}_i + 1]\hat{L}_i^i - (1-\theta_j)^2 \hat{p}_i[(1-\theta_i)\hat{p}_j + 1]\hat{L}_j^i \quad (8)$$

and

$$E(x_j) = \theta_j \hat{y}_j + (1-\theta_j)\hat{z}_j - (1-\theta_j)^2 \hat{p}_i[(1-\theta_i)\hat{p}_j + 1]\hat{L}_j^j - (1-\theta_i)^2 \hat{p}_j[(1-\theta_j)\hat{p}_i + 1]\hat{L}_i^j \quad .(9)$$

The possible strategies and expected payoffs for the partners are summarized in Table 1.

Insert Table 1 here

The top-left cell of the matrix displays the payoffs for the ethical behavior - mutual full commitment - which is also the rational strategy for both partners when the partnership is automatically renewed indefinite number of times (e.g., a Catholic marriage). In the case of a one-shot partnership, or a partnership that is automatically renewed a known finite number of times, each partner might resort to a non-ethical

[1] The first line in equation 7 displays the return for *I* in the event that *J* discovers *I*'s lack of full commitment and *I* discovers *J*'s lack of full commitment. The second line indicates the return for *I* in the event that *J* discovers *I*'s lack of full commitment and *I* does not discover *J*'s lack of full commitment. The third line presents the return for *I* in the event that *I* discovers *J*'s lack of full commitment and *J* does not discover *I*'s lack of full commitment. The fourth line displays the return for *I* in the event that both *I* and *J* do not discover each other's lack of full commitment.

behavior by rationally choosing partial commitment so long that her expected net external return exceeds her internal return:

$$\hat{z}_i - (1-\theta_i)\hat{p}_j\hat{L}_i^i > \hat{y}_i \tag{10}$$

and

$$\hat{z}_j - (1-\theta_j)\hat{p}_i\hat{L}_j^j > \hat{y}_j\,. \tag{11}$$

In this case, the entries in the bottom-right cell indicate the Nash-equilibrium payoffs.[2]

4. Nash-equilibrium partial commitment in a one-shot partnership

Assuming, for simplicity, risk-neutrality,[3] the partners' Nash-equilibrium degrees of commitment in a one-shot partnership game are those maximizing each partner expected net return simultaneously. Assuming that conditions 10 and 11 hold, the partners' chosen partial degrees of commitment satisfy:

$$(\hat{z}_i - \hat{y}_i - \hat{p}_i\hat{p}_j(1-\theta_j)^2\hat{L}_j^i) - 2\hat{p}_j[(1-\theta_j)\hat{p}_i + 1]\hat{L}_i^i(1-\theta_i^{\,o}) = 0 \tag{12}$$

and

$$(\hat{z}_j - \hat{y}_j - \hat{p}_j\hat{p}_i(1-\theta_i)^2\hat{L}_i^j) - 2\hat{p}_i[(1-\theta_i)\hat{p}_j + 1)\hat{L}_j^j(1-\theta_j^{\,o}) = 0 \tag{13}$$

which imply that *I*'s reaction function is

$$\theta_i^o = 1 - \frac{\hat{z}_i - \hat{y}_i - \hat{p}_i\hat{p}_j(1-\theta_j)^2\hat{L}_j^i}{2\hat{p}_j[(1-\theta_j)\hat{p}_i + 1]\hat{L}_i^i} \tag{14}$$

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[2] If *J* chooses to be fully committed, *I* chooses partial commitment so long that $\theta_i\hat{y}_i + (1-\theta_i)\hat{z}_i - (1-\theta_i)^2\hat{p}_j\hat{L}_i^i > \hat{y}_i$ which, by rearranging terms, is equivalently rendered by condition 10. If *I* chooses to be fully committed, *J* chooses partial commitment so long that $\theta_j\hat{y}_j + (1-\theta_j)\hat{z}_j - (1-\theta_j)^2\hat{p}_i\hat{L}_j^j > \hat{y}_j$ which, by rearranging terms, is equivalently expressed by condition 11.

[3] Risk aversion can be incorporated, with a huge computational cost, by displaying the decision problem of an R_i-type risk averse person as $\max_{\theta_i}[E(x_i) - R_i VAR(x_i)]$ and using $E(x_i^{\,2}) - (E(x_i))^2$ to compute $VAR(x_i)$.

and J's reaction function is

$$\theta_j^{\,o} = 1 - \frac{\hat{z}_j - \hat{y}_j - \hat{p}_j \hat{p}_i (1-\theta_i)^2 \hat{L}_i^j}{2\hat{p}_i[(1-\theta_i)\hat{p}_j + 1]\hat{L}_j^j}. \tag{15}$$

The equilibrium degrees of commitment $(\theta_i^{\,e}, \theta_j^{\,e})$ simultaneously satisfy equations 14 and 15. Subsequently,

$$\theta_i^e = 1 - \frac{\hat{z}_i - \hat{y}_i - \hat{p}_i \hat{p}_j \{\frac{\hat{z}_j - \hat{y}_j - \hat{p}_j \hat{p}_i (1-\theta_i^e)^2 \hat{L}_i^j}{2\hat{p}_i[(1-\theta_i^e)\hat{p}_j + 1]\hat{L}_j^j}\}^2 \hat{L}_j^i}{2\hat{p}_j\{\hat{p}_i \frac{\hat{z}_j - \hat{y}_j - \hat{p}_j \hat{p}_i (1-\theta_i^e)^2 \hat{L}_i^j}{2\hat{p}_i[(1-\theta_i^e)\hat{p}_j + 1]\hat{L}_j^j} + 1\}\hat{L}_i^i} \tag{16}$$

and

$$\theta_j^e = 1 - \frac{\hat{z}_j - \hat{y}_j - \hat{p}_i \hat{p}_j \{\frac{\hat{z}_i - \hat{y}_i - \hat{p}_j \hat{p}_i (1-\theta_j^e)^2 \hat{L}_j^i}{2\hat{p}_j[(1-\theta_j^e)\hat{p}_i + 1]\hat{L}_i^i}\}^2 \hat{L}_i^j}{2\hat{p}_i\{\hat{p}_j \frac{\hat{z}_i - \hat{y}_i - \hat{p}_j \hat{p}_i (1-\theta_j^e)^2 \hat{L}_j^i}{2\hat{p}_j[(1-\theta_j^e)\hat{p}_i + 1]\hat{L}_i^i} + 1\}\hat{L}_j^j}. \tag{17}$$ [4]

5. Numerical simulations and conclusion

Table 2 summarizes the simulation results of the model parameter effects on θ_i^e and $\theta_j^{\,e}$ obtained by using equations 16 and 17 with the benchmark set of parameter values indicated by the bold numbers in the central column. The

[4] In the special case where losses are total, rather than proportional to the degree of lack of commitment, close-form solutions are obtained. In this case, the expected return for I is $E(x_i) = \theta_i \hat{y}_i + (1-\theta_i)\hat{z}_i - (1-\theta_i)\hat{p}_j[(1-\theta_j)\hat{p}_i + 1]\hat{L}_i^i - (1-\theta_j)\hat{p}_i[(1-\theta_i)\hat{p}_j + 1]\hat{L}_j^i$, the necessary condition for maximum is $E(x_i)$ is $\hat{z}_i - \hat{y}_i - \hat{p}_i \hat{p}_j (1-\theta_j^{\,e})(\hat{L}_i^i + \hat{L}_j^i) - \hat{p}_j \hat{L}_i^i = 0$ and the Nash-equilibrium degree of commitment is $\theta_j^{\,e} = 1 - \frac{\hat{z}_i - \hat{y}_i - \hat{p}_j \hat{L}_i^i}{\hat{p}_j \hat{p}_i (\hat{L}_i^i + \hat{L}_j^i)}$ for J and, by symmetry, $\theta_i^e = 1 - \frac{\hat{z}_j - \hat{y}_j - \hat{p}_i \hat{L}_j^j}{\hat{p}_j \hat{p}_i (\hat{L}_j^j + \hat{L}_i^j)}$ for I.

benchmark parameter values portray a symmetric case ($\hat{y}_1 = \$100{,}000 = \hat{y}_2$, $\hat{z}_1 = \$200{,}000 = \hat{z}_2$, $\hat{L}_1^1 = \$100{,}000 = \hat{L}_2^2$, $\hat{L}_2^1 = \$150{,}000 = \hat{L}_1^2$ and $\hat{p}_1 = 0.9 = \hat{p}_2$) and a 0.5907 commitment of *I* and *J* to their partnership. The off-central column cells in each row report the simulation results obtained by changing the value of one parameter while holding the rest at the benchmark level. Similar results were obtained with different benchmark sets of parameters.

$\boxed{\text{Insert Table 2 here}}$

The simulations' results suggest the following properties of the partners' Nash-equilibrium degrees of commitment to their partnership. First, the commitment of each partner increases with her own internal return and diminishes with her counterpart's internal return. Second, the commitment of each partner diminishes with her external return and increases with her counterpart's external return. Third, the commitment of each partner increases with her own quitting cost and decreases with her counterpart's quitting cost. Fourth, the commitment of each partner increases with her own costs of being deserted and decreases with her counterpart's costs of being deserted. Fifth, the commitment of each partner decreases with her own intrinsic capacity to detect her counterpart's lack of full commitment and increases with her counterpart's intrinsic capacity to detect lack of full commitment.

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Table 1. The partners' expected payoff matrix

1→ 2 ↓	Full Commitment	Partial Commitment
Full Commitment	$E(x_1)=\hat{y}_1$ $E(x_2)=\hat{y}_2$	$E(x_1)=\theta_1\hat{y}_1+(1-\theta_1)\hat{z}_1 - (1-\theta_1)^2\hat{p}_2\hat{L}_1^1$ $E(x_2)=\hat{y}_2-(1-\theta_1)^2\hat{p}_2\hat{L}_1^2$
Partial Commitment	$E(x_1)=\hat{y}_1-(1-\theta_2)^2\hat{p}_1\hat{L}_2^1$ $E(x_2)=\theta_2\hat{y}_2+(1-\theta_2)\hat{z}_2 - (1-\theta_2)^2\hat{p}_1\hat{L}_2^2$	$E(x_1)=\theta_1\hat{y}_1+(1-\theta_1)\hat{z}_1 - (1-\theta_1)^2\hat{p}_2[(1-\theta_2)\hat{p}_1+1]\hat{L}_1^1 - (1-\theta_2)^2\hat{p}_1[(1-\theta_1)\hat{p}_2+1)\hat{L}_2^1$ $E(x_2)=\theta_2\hat{y}_2+(1-\theta_2)\hat{z}_2 - (1-\theta_2)^2\hat{p}_1[(1-\theta_1)\hat{p}_2+1]\hat{L}_2^2 - (1-\theta_1)^2\hat{p}_2[(1-\theta_2)\hat{p}_1+1)\hat{L}_1^2$

Table 2. Numerical simulation results

$\hat{y}_1$	60,000	80,000	**100,000**	120,000	140,000
θ_1^e	*0.4115*	*0.5011*	*0.5907*	*0.6803*	*0.7699*
θ_2^e	*0.6833*	*0.6354*	*0.5907*	*0.5465*	*0.5038*
$\hat{y}_2$	60,000	80,000	**100,000**	120,000	140,000
θ_1^e	*0.6833*	*0.6354*	*0.5907*	*0.5465*	*0.5038*
θ_2^e	*0.4115*	*0.5011*	*0.5907*	*0.6803*	*0.7699*
$\hat{z}_1$	160,000	180,000	**200,000**	220,000	240,000
θ_1^e	*0.7699*	*0.6803*	*0.5907*	*0.5011*	*0.4115*
θ_2^e	*0.5038*	*0.5465*	*0.5907*	*0.6364*	*0.6833*
$\hat{z}_2$	160,000	180,000	**200,000**	220,000	240,000
θ_1^e	*0.5038*	*0.5465*	*0.5907*	*0.6364*	*0.6833*
θ_2^e	*0.7699*	*0.6803*	*0.5907*	*0.5011*	*0.4115*
$\hat{L}_1^1$	50,000	75,000	**100,000**	125,000	150,000
θ_1^e	*0.1815*	*0.4543*	*0.5907*	*0.6726*	*0.7272*
θ_2^e	*0.7545*	*0.6438*	*0.5907*	*0.5597*	*0.5395*
$\hat{L}_2^1$	100,000	125,000	**150,000**	175,000	200,000
θ_1^e	*0.5788*	*0.5843*	*0.5907*	*0.5972*	*0.6036*
θ_2^e	*0.6137*	*0.6022*	*0.5907*	*0.5794*	*0.5681*
$\hat{L}_2^2$	50,000	75,000	**100,000**	125,000	150,000
θ_1^e	*0.7545*	*0.6438*	*0.5907*	*0.5794*	*0.5681*
θ_2^e	*0.1815*	*0.4543*	*0.5907*	*0.5972*	*0.7272*
$\hat{L}_1^2$	100,000	125,000	**150,000**	175,000	200,000
θ_1^e	*0.6137*	*0.6022*	0.5907	*0.5794*	*0.5681*
θ_2^e	*0.5788*	*0.5843*	0.5907	*0.5972*	*0.6036*
$\hat{p}_1$	0.8000	0.8500	**0.9000**	0.9500	0.9999
θ_1^e	*0.6036*	*0.5969*	*0.5907*	*0.5850*	*0.5796*
θ_2^e	*0.5492*	*0.5712*	*0.5907*	*0.6081*	*0.6237*
$\hat{p}_2$	0.8000	0.8500	**0.9000**	0.9500	0.9999
θ_1^e	*0.5492*	*0.5712*	*0.5907*	*0.6081*	*0.6237*
θ_2^e	*0.6036*	*0.5969*	*0.5907*	*0.5850*	*0.5796*